## SECONDARY MATHEMATICS TEACHERS' ASSOCIATION

(SMATA)


# S. 4 SEMINAR 2019 

THE $4^{\text {th }}$ ANNUAL GRAND SMATA MATHEMATICS ZONAL SEMINAR ORGANIZED

AT<br>ST. JOSEPH OF NAZARETH HIGH SCHOOL SATURDAY 8 ${ }^{\text {th }}$, June 2019<br>O'LEVEL MATHEMATICS PAPER I \& II<br>DISTRIBUTION OF TOPICS



## SEMINAR QUESTIONS 2019

## GENERAL QUESTIONS

1. Express $1.21555 \ldots$ as a fraction in the form $p \frac{q}{r}$. Hence, find the value of $(r-q)$.
2. Evaluate: $\left(\frac{9}{4}\right)^{\frac{1}{2}} \times\left(\frac{81}{16}\right)^{\frac{-3}{4}} \times\left(\frac{1}{27}\right)^{\frac{-2}{3}}$.
3. The operation is defined by $a * b=a(b+4)$. Solve the equation $2 * x=6 * 2$.
4. Six men take 10 days to plant 480 pine trees. Find how many pine trees 9 men can plant in 8 days.
5. The figure below shows a compound plan to be used in a certain hotel gardens.


If the shaded part of the compound is to be filled with flowers. Find the area they will occupy.
6. Simplify $2 \sqrt{50}+4 \sqrt{2}+\sqrt{18}-7 \sqrt{18}$.
7. If $y=a\left(\sqrt[3]{\frac{x^{2}-n}{m}}\right)$. Make $x$ the subject of the formula.
8. Given that vectors $a=\binom{\mathbf{3}}{-2}, b=\binom{-\mathbf{1}}{-\mathbf{4}}$ and $\boldsymbol{c}=\binom{\mathbf{4}}{-\mathbf{1 2}}$.

Find (i) the modulus of $(\boldsymbol{a}+\boldsymbol{b})$
(ii) the value of scalar $\boldsymbol{k}$ such that $\boldsymbol{k}(\boldsymbol{a}+\boldsymbol{b})=\boldsymbol{c}$.
9. Find all the integral values of $x$ which satisfy the inequalities below;

$$
2(2-x)<4 x-9<x+11
$$

10. Without using mathematical tables or calculator evaluate:
$2 \log _{10} 40-\frac{1}{4} \log _{10} 256+2 \log _{10} 5$

## MATRICES

11. Given that $\mathrm{A}=\left(\begin{array}{ll}1 & 2 \\ 1 & 0\end{array}\right), \quad \mathrm{B}=\left(\begin{array}{ll}6 & 3 \\ 0 & 9\end{array}\right)$ and $\mathrm{C}=\left(\begin{array}{cc}3 & -2 \\ 1 & 2\end{array}\right)$,

Find (i) the matrix P , such that $\mathrm{P}=2 A-\frac{1}{3} B+C^{2} \quad$ (ii) $\mathrm{P}^{-1}$, the inverse of P .
12. (a) Find the inverse of $\left(\begin{array}{cc}3 & 4 \\ 1 & -2\end{array}\right)$. Hence, solve the simultaneous equation

$$
\begin{aligned}
3 x+4 y & =1 \\
x & =2 y
\end{aligned}
$$

(b) If $\left(\begin{array}{cc}x & 3 \\ 2 & x-5\end{array}\right)$ is a singular matrix, solve for $x$.
13. Two university students went to a Freshers' Bazar. John bought 2 kg of Sugar, 3 loaves of bread and 2 litres of milk. Martha bought 5 kg of sugar, 2 loaves of bread, 3 litres of milk and 4 kg of rice.
The prices of commodities were as follows:
Sugar shs. 5000 per kg; Bread shs. 4200 per loaf; Milk shs. 1400 per litre and Rice shs. 3800 per kg.
(a) From the above information write down;
(i) $\mathrm{a} 2 \times 4$ matrix for the purchases
(ii) a column matrix for the prices.
(b) Use the matices in (a) (i) and (ii) above, to calculate the total expenditure of each person. Hence, determine the difference in their expenditure.

## INDICES, LOGARITHMS AND SURDS

14. Solve the equation: $\frac{2^{2 x}}{16^{(x-1)}} \times 8^{x-1}=64$
15. Use logarithm tables to evaluate:
$\left(\frac{14.26 \times 0.032}{0.006}\right)^{\frac{2}{3}}$
16. Simplify $(1+\sqrt{2})(1-\sqrt{2})$. Hence or otherwise, evaluate $\frac{2}{1+\sqrt{2}}$ to 3 significant figures. Given that $\sqrt{2}=1.4142$.

## QUADRATICS

17. (a) Draw the graph of $y=(2 x-1)(x+3)$ for $-4 \leq x \leq 3$.
(b) Use your graph to solve the equations;
(i) $(2 x-1)(x+3)=0$
(ii) $2 x^{2}+5 x-7=0$.
(c) From the graph, find the solution of the inequality: $2 x^{2}+5 x-7<0$

## SET THEORY AND LOGIC

18. A group of fans of a certain Radio Station in Kampala were asked the kind of programs they like listening to. It was found out that 32 like Politics (P), 40 like Health (H), while 28 like Development (D). 17 fans like both Politics and Health, 12 like both Politics and Development, and 18 like Health and Development, while 5 like none of the three programs. It was also found out that the number of fans who like both Politics and Development is twice those who like both Politics and Health.
(a) Draw a venn-diagram to represent the above information.
(b) Use the Venn- diagram in (a) to find the number of fans who like;
(i) All the radio programs.
(ii) Only one radio programs.
(c) Calculate;
(i) The total number of fans in the group.
(ii) The percentage of fans who like at most two programs.
(d) Find the probability that a fan chosen at random from the group likes;
(i) Only one radio program
(ii) At least two programs

## STATISTICS

19. In an agricultural research centre, the heights of a sample of 50 banana suckers were measured and recorded as shown in the table below.

| Height (cm) | Cumulative frequencies of suckers |
| :---: | :---: |
| $30-34$ | 3 |
| $35-39$ | 15 |
| $40-44$ | 25 |
| $45-49$ | 33 |
| $50-54$ | 43 |
| $55-59$ | 49 |
| $60-64$ | 50 |

(a) Draw an Ogive and use it to estimate;
(i) The median height
(ii) $80^{\text {th }}$ percentile.
(b) Using a working mean of 47 cm , calculate the exact mean height of the suckers.

## COMMISSION, INTEREST AND HIRE PURCHASE

20. (a) The cost of a car outside Uganda is US $\$ 5,000$. For one intending to buy such car through an agent who deals in Japanese Yen, the agent will charge you $20 \%$ commission on the price of the car and a further 80,350 Japanese Yen for shipment of the car.
Given that;
1 US \$ = 112.00 Yen
1 US \$ = Ugshs.3,800
Calculate how much money in Uganda shillings would one need to send to the agent in Japan to obtain the car.
21. Twins, Waswa and Kato each borrowed Shs. 200,000 from a certain commercial bank for a loan period of 2 years. Given that Waswa opted for simple interest while Kato chose compound interest system. If the bank charges a uniform interest rate of $15 \%$ per annum.
Find; (i) the amount each returned to the bank.
(ii). who paid back more money than the other, and by how much?
22. The following is an advertisement for a Laptop on sale.

## BRAND NEW LAPTOPS

## TERMS:

VALUE: Shs. 1200,000
CASH : 2\% Discount of the value.

## HIRE PURCHASE:

Deposit 10\% of the value and pay either; Shs. 50,000 weekly for 26 weeks or Shs. 200,000 for 6 months.
(a) Calculate the amount of money one would pay for the laptop on weekly hire purchase.
(b) Determine the Savings one would make by buying the Laptop on cash terms rather than on monthly hire purchase.

## PROBABILITY

23. A letter is chosen at random from the word 'SUBTRACTION'. What is the probability that a letter is;
(i) a vowel
(ii) a consonant.
24. A bag contains 7 mangoes, some apples and tomatoes. The probability of picking an apple from the bag is $\frac{1}{4}$. While that of picking a tomato from the bag is $\frac{2}{5}$. Determine the number of apples and tomatoes in the bag.
25. A bag contains 4 apples and 3 mangoes. Two fruits are picked from the bag one at ago without replacement. Find the probability that;
(i) Both are apples that are picked
(ii) One apple and one mango is picked.
26. A fair die and a coin are respectively thrown and tossed once. Find
(i) The probability space
(ii) The probability of obtaining a tail and an odd number.

## FUNCTIONS

27. Given that $f(\mathrm{x})=\frac{1}{x-5}$

Find:
(i) $\quad f(-1)$
(ii) $f^{-1}(2)$
(iii) The value of $x$ for which $f(x)$ is undefined.
28. Given that $f(x)=2 x+3, g(x)=4 x-1$.

Find (i) $f^{-1}(1) \quad$ (ii) $[f \circ g](x) \quad$ (iii) $g f(4) \quad$ (iv) $f f(x)$
29. If $h(x)=a x^{2}-b x, h(2)=8$ and $h(-1)=5$. Find the value of $a$ and $b$.

## TRIGONOMETRY

30. A telecom signal antenna stands directly across the street from a building. The angle of depression of the top of the building from the top of the antenna is $30^{\circ}$ and the angle of elevation of the top of the antenna from the foot of the building is $44^{\circ}$. Given that the distance between the antenna and the building is 40 m , calculate to two decimal places:
(a) The height of the antenna.
(b) The difference in height between the antenna and the building.
(c) The height of the building.
(d) The angle of elevation of the top of the building from the foot of the antenna.

## LINEAR EQUATIONS

31. Solve the equation: $\frac{x-3}{3}-\frac{x+2}{4}=\frac{x}{2}-\frac{1}{3}$
32. Byanyima is four times as old as Kamugisha. Three years ago the sum of their ages was 34 . Find their present age.

## THREE DIMENSIONS (3D SHAPES)

33. The figure below shows a water tank on a farm in form of a right pyramid mounted on a cuboid. $A B=B C=15 \mathrm{~m}, C G=8 \mathrm{~m}$ and $V G=17 \mathrm{~m}$.


Calculate;
(a) (i) the length of line AC
(ii) the length of projection of line BH on the plane $A B C D$.
(b). The angle between the line AG and the plane ABCD.
(c) the vertical height of the point V from the plane ABCD .
(d) the angle between;
(i) the plane VFE and ABCD.
(ii) the plane VFG and VEH.
(iii). The plane BCGF and BCHE.
(e) Calculate the volume of the tank.

## LINEAR PROGRAMMING

34. A company is considering installing two types of machines, A and B. The information about each type of machine is given in the table below.

| Machine type | Number of operators | Floor space (m²) | Daily profit(shs) |
| :---: | :---: | :---: | :---: |
| A | 2 | 5 | 15,000 |
| B | 5 | 8 | 25,000 |

The company decided to install $x$ machines of type $A$ and $y$ machines of type $B$.
(a) Write down the inequalities that express the following conditions:
(i) The number of operators available is 40.
(ii) The floor space available is $80 \mathrm{~m}^{2}$.
(iii) The company is to install not less than 3 of type A machines.
(iv) The number of type B machines must be more than one third the number of type A machines.
(b) On the same graph, draw the inequalities in part (a), above and shade the unwanted regions.
(c) Draw a search line and use it to determine the number of machines of each type that should be installed to maximize the daily profit.

## TAXATION

35. The tax structure of taxable income of employees of different companies in a certain country is as follows:

| Taxable income (Ug shs) | Tax rates (\%) |
| :---: | :---: |
| $0-210,000$ | free |
| $210,001-310,000$ | 10.5 |
| $310,001-500,000$ | 20.0 |
| $500,001-600,000$ | 35.0 |
| 600,001 and above | 40.0 |

Mr. Kasolo is an employee of a certain company. He earned a gross monthly income of shs. 980,000 in March 2019. Given that he is entitled to the following allowances;

Transport
Housing
Medical
Insurance premium
Lunch
shs. 624,000 p.a
shs 200,000 per months.
$\frac{1}{20}$ th of gross monthly income.
shs. 17,000 per month.
shs. 3,000 per day.

Calculate his;
(a) taxable income
(b) income tax
(c) net income
(d) Express income tax he pays a year as a percentage of his annual gross income.

## CIRCLE PROPERTIES

36. The figure shows a circle centre, $O$ and a cyclic quadrilateral $P Q R S$. $P R=R Q$, angle $P R Q=70^{\circ}$ and $Q O S$ is a straight line.


Find the size of the following angles, giving reason for your answer;
(i) angle PRS
(ii) angle POQ
(iii) angle RPS
(iv) angle PSR
(v) reflex angle POQ

## MENSURATION

37. The figure below shows a model of a solid tower to be put on a Five Star Hotel in Kampala city. $\left(\right.$ Take $\left.=\frac{22}{7}\right)$.

(a) Find the total curved surface area of the model.
(b) Determine the volume of the model
(c) The model is to be painted (excluding the base) with a coat of paint that is 1 mm thick. How many litres of paint will be required?
(d) Given that the cost of painting a square metre ( $1 \mathrm{~m}^{2}$ ) of the actual tower is shs. 12,500. Find how much money is needed to paint the tower, if the architectual plan of the tower has a scale of $1: 100$.

## BEARING

38. Four towns $A, B, C$ and $D$ are situated on the same horizontal ground such that town $B$ is 200 km on a bearing of $N 65^{\circ} \mathrm{E}$ from town A. Town C is 300 km on a bearing of $120^{\circ}$ from $B$ and town $D$ is 150 km due west of town $C$.
(a) Draw a sketch diagram showing the relative positions of the four towns.
(b) Using a suitable scale, draw an accurate drawing representing the positions of towns A, B, C and D.
(c) Determine the distance and bearing of;
(i) Town D from A
(ii) Town B from D.

## CONSTRUCTION

39. Using a ruler, pencil and a pair of compasses only.
(a) Construct a triangle PQR in which $\mathrm{PQ}=11.5 \mathrm{~cm}, \angle P Q R=60^{\circ}$ and $\mathrm{QR}=8.4 \mathrm{~cm}$.
(b). Construct the bisector of angles PRQ and QPR. Let them meet at point S.
(c) From S , draw a perpendicular to PQ meeting PQ at T .
(i) Measure ST. Hence, find the area of the circle.
(ii) State how far is the point $S$ from $P R$ and $Q R$.

## VECTORS

40. The figure shows a triangle $O A B$ in which $O A=\boldsymbol{a}$ and $O B=\boldsymbol{b} . \mathrm{M}$ and N are points on $O B$ and $A B$ respectively such that $O M=\frac{1}{3} O B$ and $A N=\frac{2}{5} A B$. Lines $A M$ and $O N$ meet at $P$ such that $O P=\frac{5}{9} O N$

(a) Express the following vectors in terms of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$;
(i) $\boldsymbol{A B}$
(ii) $\mathbf{O N}$
(iii) $\boldsymbol{A M}$
(b) Express $\boldsymbol{A P}$ and $\boldsymbol{P} \boldsymbol{M}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. Hence, show that the points $\mathrm{A}, \mathrm{P}$ and $M$ are collinear.
(c) State the ratio $\mathrm{AP}: \mathrm{PM}$.

## TRANSFORMATION MATRICES

41. A triangle $P Q R$ has vertices $P(4,2), Q(1,1)$ and $R(5,1)$.
(a) Plot the triangle PQR on the grid.
(b) Given that $P^{\prime} Q^{\prime} R^{\prime}$ is the image of $P Q R$ under an enlargement of scale factor -1 centre $(-1,-1)$. State the coordinates of $P^{\prime}, Q^{\prime}$ and $R^{\prime}$.
(c) $P^{\prime \prime} Q^{\prime \prime} R^{\prime \prime}$ is the image of $P Q R$ under a translation $T=\binom{1}{-3}$. Locate $P^{\prime \prime} Q^{\prime \prime} R^{\prime \prime}$ on the graph and state the coordinates of $P^{\prime \prime}, Q^{\prime \prime}$ and $R^{\prime \prime}$.
(d) $P^{\text {III }} Q^{\prime \prime \prime} R^{I I I}$ is the image of $P Q R$ under a reflection in the line $x=-1$. Determine the the coordinates of $P^{I I I}, Q^{I I I}$ and $R^{\text {III }}$.
(e) $P_{4} Q_{4} R_{4}$ is the image of $P Q R$ under a rotation of positive quarter turn about the origin. State the coordinates of $\mathrm{P}_{4}, \mathrm{Q}_{4}$ and $\mathrm{R}_{4}$.

## KINEMATICS

42. The distance between Odongo's home and Akena's house is 40 km . One day, Odongo left his home at 9:00 am and cycled towards akena's house at an average speed of $20 \mathrm{~km} / \mathrm{hr}$. After $1 \frac{1}{2}$ hours, Akena also left his house and cycled using the same road towards Ondongo's home at an average speed of $30 \mathrm{~km} / \mathrm{hr}$.
(a) Determine:
(i) The distance from Odongo's home where the two cyclists met.
(ii) The time of the day when they met.
(b) The two continued after meeting with their respective journeys until each reached his destination. Determine who reached his destination earlier and by how long.

## COORDINATE GEOMETRY

43. Given that points $A(-1,2)$ and $B(5,4)$ lie on a straight line $A B$. Find the;
(i) Length of line $A B$
(ii) Equation of AB
(iii) Equation of the perpendicular bisector of $A B$.
44. The figure below shows two lines $y=3 x$ and $x+y=12$. Use it to determine the coordinates of points $P, Q$ and $R$

45. A line which joins the points $S(3, k)$ and $T(-2,5)$ is parallel to another line whose equation is $2 x+5 y=10$. Find the value of $k$.

## RATIOS , PROPORTIONS AND VARIATIONS

46. A squatter has a plot of land which covers an area of $\mathbf{6} \mathbf{c m}^{2}$ on a map whose scale is $1: 50,000$. Find the actual area of the squatter's land in $\mathrm{km}^{2}$.
47. Given that $x: y=3: 4$ and $x+y=7$, find the value of $2 x-y$.
48. The distance (d) in metres travelled by a certain car being brought to rest from a speed of $\mathrm{V} \mathrm{ms}{ }^{-1}$ by application of brakes partly varies as the speed, V and partly as the square of the speed. Given that when $\mathrm{d}=62 \mathrm{~m}, \mathrm{~V}=40 \mathrm{~ms}^{-1}$ and when $\mathrm{d}=117 \mathrm{~m}, \mathrm{~V}=60 \mathrm{~ms}^{-1}$.
(i) Form an equation relating d and V .
(ii) If the brakes are applied when the car's speed is $50 \mathrm{~ms}^{-1}$, find the distance it travels before coming to rest.

## THEEND <br> Thank you for your participation "SMATA" <br> Together for Marthematics smatauganda@gmail.com <br> 0702790079 / 0775536151

