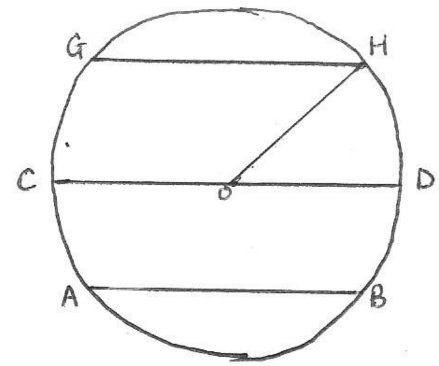
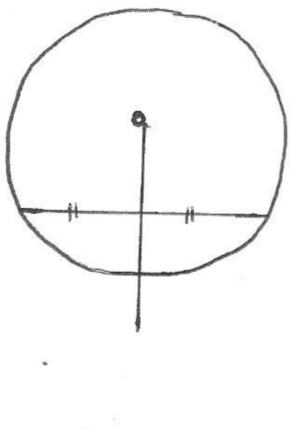
CIRCLE PROPERTIES.

Given the circle below

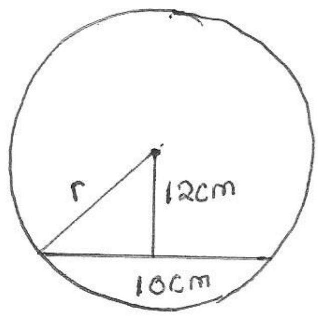


The line CD is the diameter of the circle.
The line OH from the centre of the circle to the circumference of the circle is a radius.
The lines AB and GH are called chords.

NB: A perpendicular bisector of any chord of the circle passes through the centre of the circle and will bisect the chord.

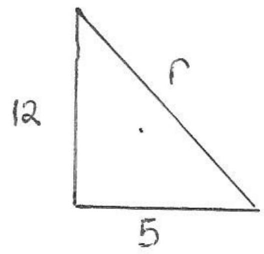


eg 1. A chord of length 10cm is at a distance of 12cm from the centre of the circle. Find the radius of the circle.



NB:
The line from the ~~circle~~ centre of the circle bisects the chord.

Therefore to find the radius we are using pythagorus theorem.



$r^2 = 5^2 + 12^2$

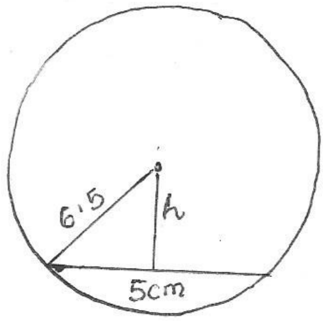
$r^2 = 25 + 144$

$r^2 = 169$

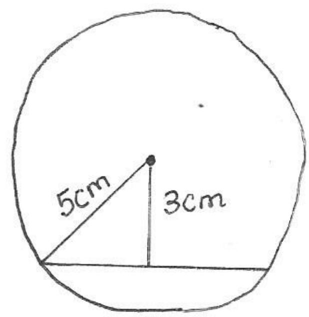
$r = 16$

$\therefore$ radius is 13cm

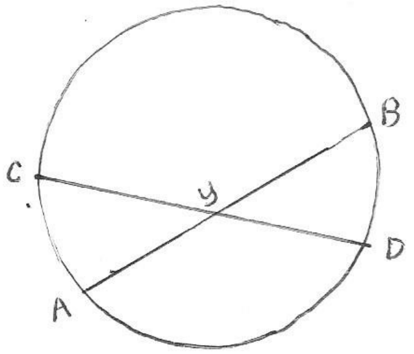
Eg 2: How far is a chord of length 5cm from the Centre of a circle of radius 6.5cm?



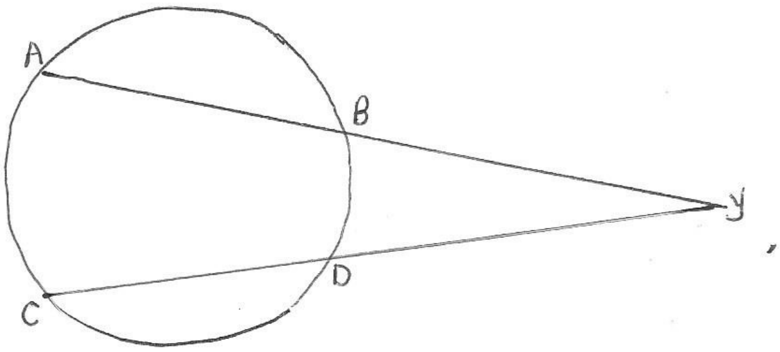
Eg 3. What is the length of a chord 3cm from the centre of a circle of radius 5cm?



INTERSECTING CHORDS (WITHIN & OUTSIDE)



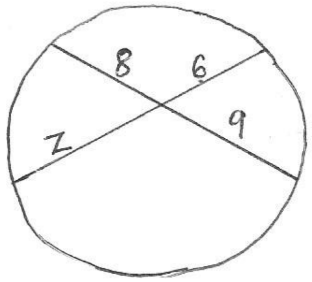
Given that the chords AB and CD are intersecting at y then $Ay \times By = Cy \times Dy$



Given that chords AB and CD are intersecting at y then
$Ay \times By = Cy \times Dy$

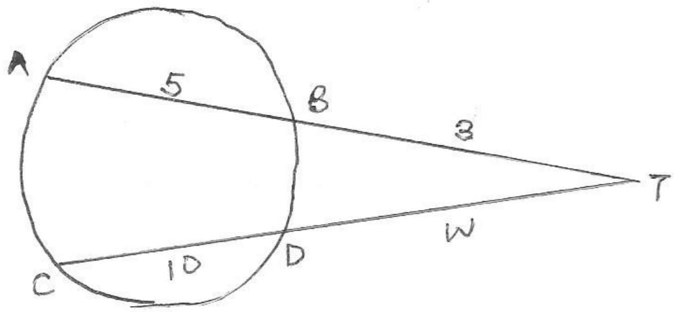
Examples

1. Find the length Z in the diagram.



$$6 \times Z = 8 \times 9$$
$$6Z = 72$$
$$Z = 12$$

2. Find the length w in the diagram.

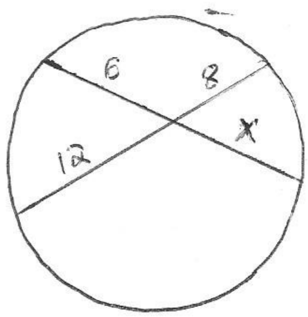


Using $AT \times BT = CT \times DT$

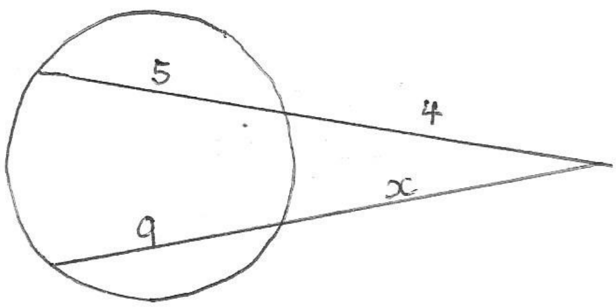
$$8 \times 3 = (10+w)w$$
$$24 = w^2 + 10w$$
$$w^2 + 10w - 24 = 0$$
$$w^2 + 12w - 2w - 24 = 0$$
$$w(w+12) - 2(w+12) = 0$$
$$(w-2)(w+12) = 0$$
$$w - 2 = 0 \text{ or } w + 12 = 0$$
$$w = 2 \text{ or } w = -12$$
$$\therefore w = 2$$

Let's try these.

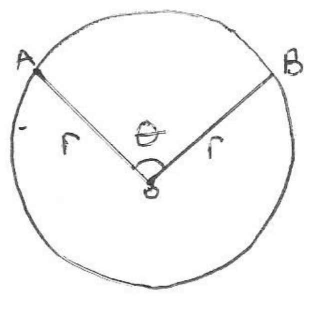
1. Find the length x



2. Find the value of x.



LENGTH OF AN ARC.



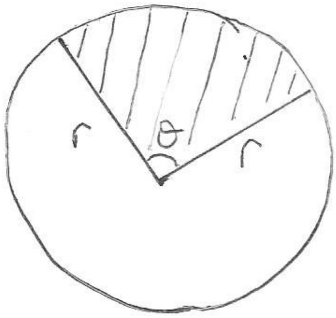
~~The~~ An arc is part of a circle

$\therefore$ the length of arc AB

$$= \frac{\theta}{360} \times 2\pi r$$

AREA OF A SECTOR

A sector is part of a circle bound by 2 radii and an arc.



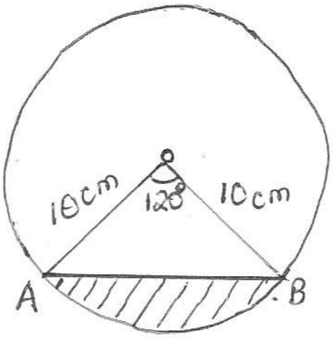
Area of a sector $= \frac{\theta}{360} \times \pi r^2$

NB: The minor arc makes a smaller angle at the centre of the circle while the major arc makes a bigger angle at the centre of the circle.

Example

An arc AB makes an angle of $120°$ at the centre of a circle. If the radius of the circle is 10 cm, calculate

(i) the length of the minor arc.
(ii) the length of the major arc
(iii) The area of the shaded part.



(i) length of the minor arc

$= \frac{\theta}{360} \times 2\pi r$

$= \frac{\cancel{120}^{1}}{\cancel{360}_{3}} \times 2 \times 3.14 \times 10$

$= 20.93$ cm

(ii) the length of the major arc.

The angle the major arc makes at the centre is $360° - 120° = 240°$

$\therefore$ the length of the major arc $= \frac{\cancel{240}^{2}}{\cancel{360}_{3}} \times 2 \times 3.14 \times 10$

$= 41.87$ cm

(iii) Area of the shaded part = Area of the Sector − Area of the triangle.

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times a \times b \sin\theta$$

$$= \frac{120}{360} \times 3.14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10 \times \sin 120$$

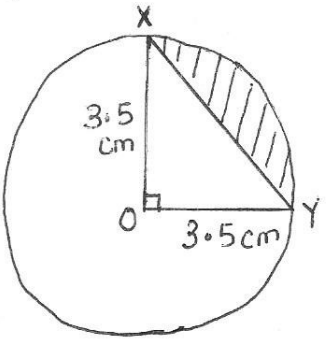
$$= \frac{\overset{1}{\cancel{120}}}{3\,\cancel{360}} \times 3.14 \times 10 \times 10 - \frac{1}{\underset{1}{\cancel{2}}} \times \overset{5}{\cancel{10}} \times 10 \times 0.8660$$

$$= 104.67 - 43.3$$

$$= 61.37\ cm^2$$

Exercise

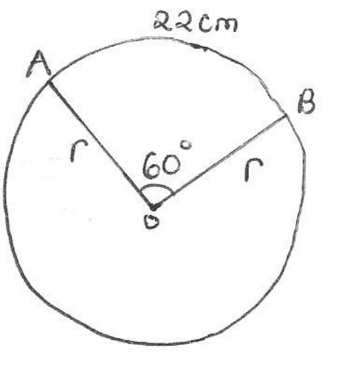
1. OX and OY are perpendicular radii of a circle of centre O. ~~with~~ of radius 3.5cm. Find the area of the shaded part.



2. The length of an arc that subtends an angle of 60° at the centre of the circle is 22cm.

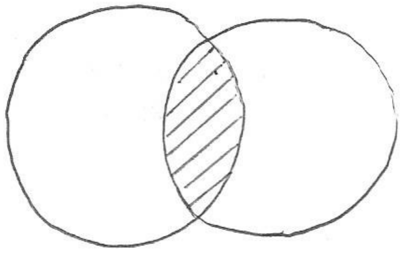
a) Find the radius of the circle.

b) Calculate the area of the sector. (use $\pi = 22/7$)

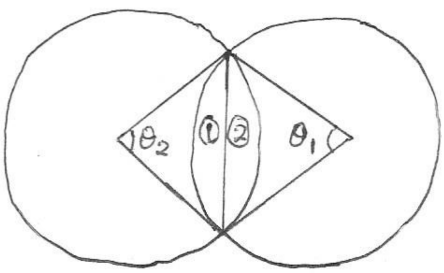


INTERSECTING CIRCLES

When circles intersect, there is a common area of intersection.



This area is calculated by adding the areas of the two segments which comprise the common area.

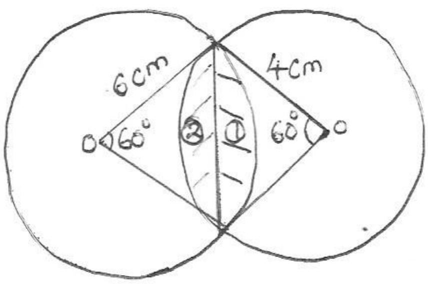


Area 1 = Area of sector with θ_1 − Area of triangle

Area 2 = Area of Sector with θ_2 − Area of triangle.

Example 1

Calculate the area of the shaded part in the circles below.



Area ① = Area of sector − Area of triangle

$$= \cancel{\frac{60}{360} \times 6 \times 6}$$

$$= \frac{\theta}{360} \times \pi r^2 \quad - \quad \frac{1}{2} ab \sin\theta$$

$$= \frac{\cancel{60}^{1}}{\cancel{360}_{6}} \times 3.14 \times 6 \times \cancel{6}^{1} \quad - \quad \frac{1}{\cancel{2}_{1}} \times \cancel{6}^{3} \times 6 \times \sin 60^\circ$$

$$= 3.14 \times 6 \quad - \quad 3 \times 6 \times 0.8660$$

$$= 18.84 \quad - \quad 15.588$$

$$= 3.252 \text{ cm}^2.$$

$$\text{Area } ② = \text{Area of sector} - \text{Area of triangle}$$

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times ab \sin\theta$$

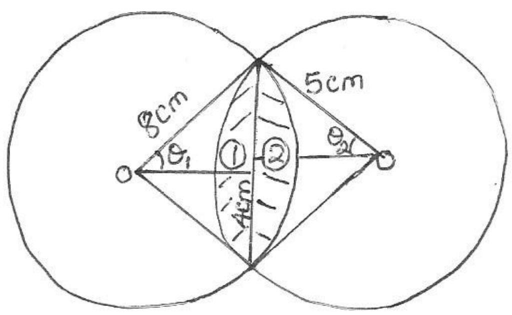
$$= \frac{\cancel{60}^{1}}{\cancel{360}_{\cancel{6}\,3}} \times 3.14 \times \cancel{4}^{2} \times 4 - \frac{1}{\cancel{2}_{1}} \times \cancel{4}^{2} \times 4 \times \sin 60^\circ$$

$$= 8.373 - 6.928$$

$$= 1.445 \text{ cm}^2$$

$\therefore$ Area of shaded part $= 3.252 + 1.445$
$= 4.697 \text{ cm}^2$.

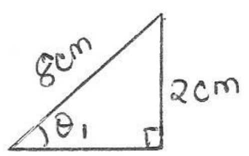
Example 2. The diagram below shows two intersecting circles of radii 5cm and 8cm, with a common chord of 4cm.



Find the area of the shaded part.

In the circles above the angles at the centres of the circles are not given. There fore we need to find the angles first.

Let's first find θ_1 using pythagorus theorem



$$\sin\theta_1 = \frac{opp}{hyp}$$

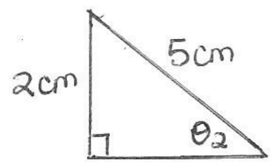
$$\sin\theta_1 = \frac{2}{8}$$

$$\sin\theta_1 = 0.25$$

$$\theta_1 = 14.48^\circ$$

$\therefore$ The angle at the centre $= 2 \times 14.48^\circ$
$= 28.96^\circ$

To find θ_2 we use



$$\sin \theta_2 = \frac{opp}{hyp}$$

$$\sin \theta_2 = \frac{2}{5}$$

$$\sin \theta_2 = 0{\cdot}4$$

$$\theta_2 = 23{\cdot}58^\circ$$

$\therefore$ The angle at the centre $= 2 \times 23{\cdot}58^\circ$

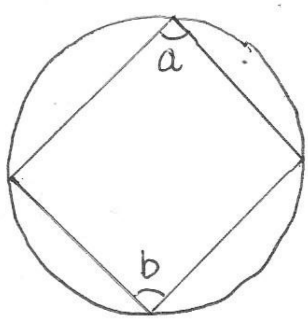
$= 47.16^\circ$

There after find the area of the shaded part. (Complete the number).

CIRCLE PROPERTIES.

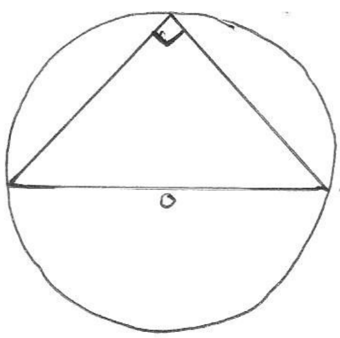
1. Opposite angles in a cyclic quadrilateral are supplementary.

<u>NB</u> A cyclic quadrilateral is a four sided figure circumscribed by a circle. ~~and~~ ie all the edges of the figure must touch the circumference of the circle.

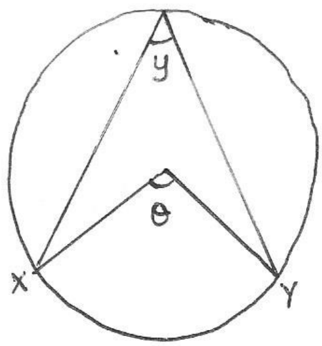


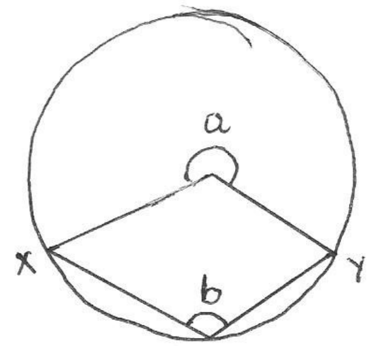
$$a + b = 180^\circ$$

2. The angle in a semi circle is a right angle.



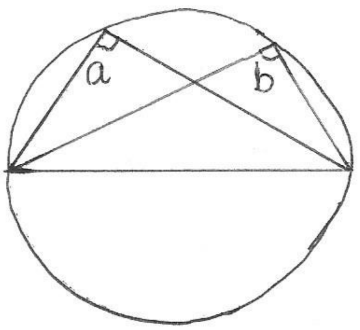
3. The angle an arc subtends at the centre is twice that it subtends at the circumference of the circle.





$\theta = 2y$ $\qquad$ $a = 2b$

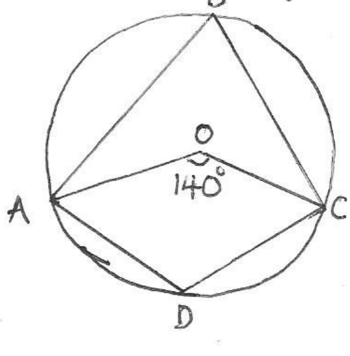
4. Angles in the same segment are equal.



$a = b$

<u>Example</u>

In the figure below O is the centre of the circle and angle AOC $= 140°$



Find (i) angle ABC

$\angle ABC = \frac{1}{2} \times AOC$ (The angle an arc subtends at the centre is twice that it subtends at the circumference)

$= \frac{1}{2} \times 140°$

$= 70°$

(ii) Angle ADC

$\angle ADC = 180^\circ - \angle ABC$ (opp. angles in a cyclic quadrilateral are supplementary)

$= 180^\circ - 70^\circ$

$= 110^\circ$

OR If we find the angle the major arc makes at the centre ie $360^\circ - 140^\circ$

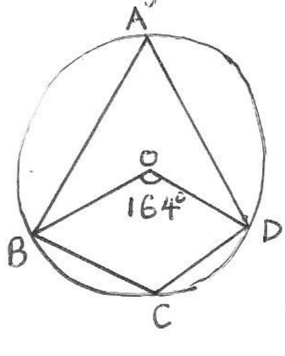
$= 220^\circ$

then using the property that the angle an arc subtends at the centre is twice that it subtends at the circumference, $\angle ADC = \frac{1}{2} \times 220^\circ$

$= 110^\circ$

Exercise

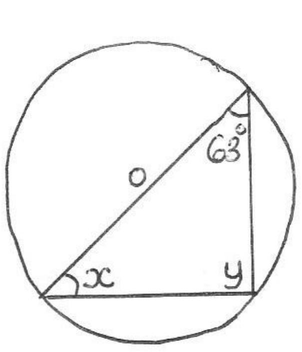
1. In the diagram below O is the centre of the circle and angle BOD $= 164^\circ$.



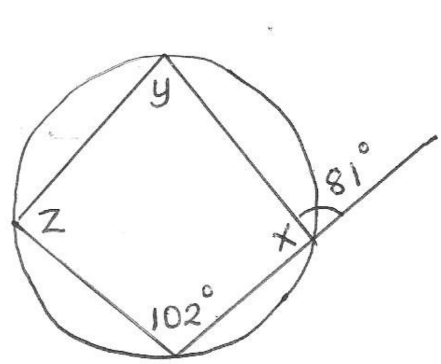
Find a) angle BAD
b) angle BCD

2. Find the value of angles marked with a letter.

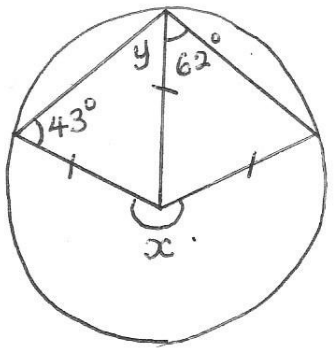
a)



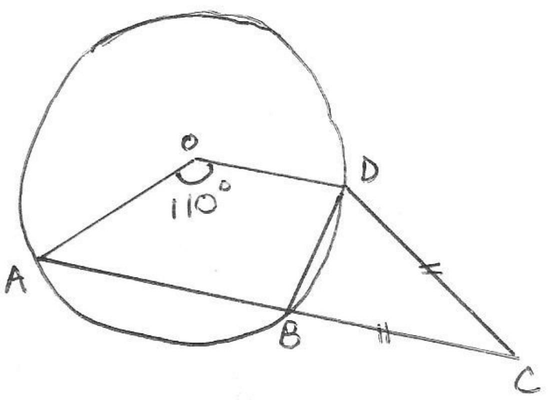
b)



C

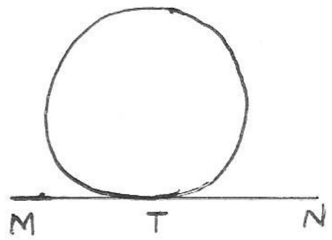


3. In the diagram below O is the centre of the circle and ABC is a straight line. BC = CD and AOD = 110°.
Find (i) DBC
(ii) BDC



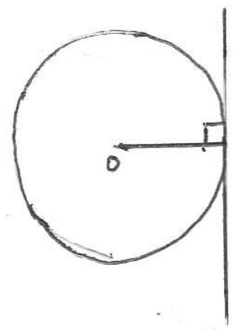
TANGENT PROPERTIES

A tangent to a circle is a line that touches the circle but does not cut through it ie

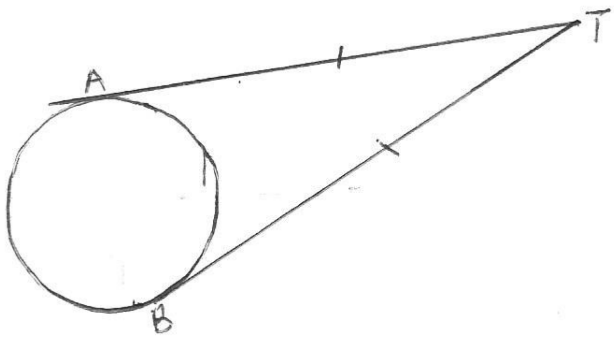


The following are the tangent properties

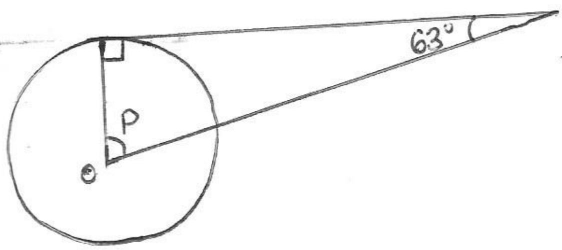
1. A tangent to a circle is perpendicular to the radius of the circle from its point of contact with the circle.



2. Tangents from an external point are equal.



Eg. 1 Find the size of the angles labelled

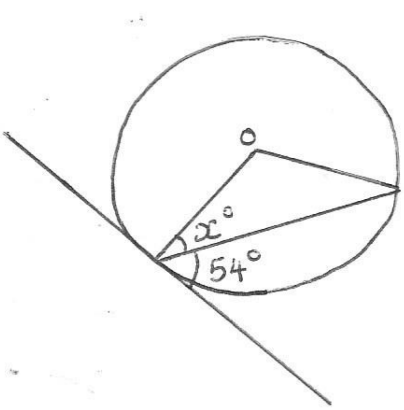


$p + 63 + 90° = 180°$

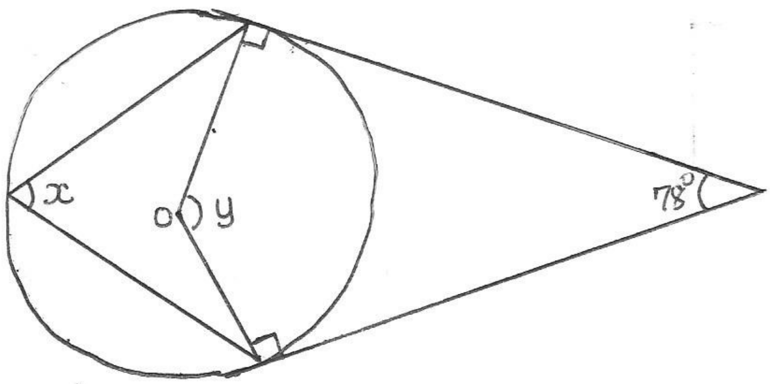
$p + 153° = 180°$

$p = 27°$

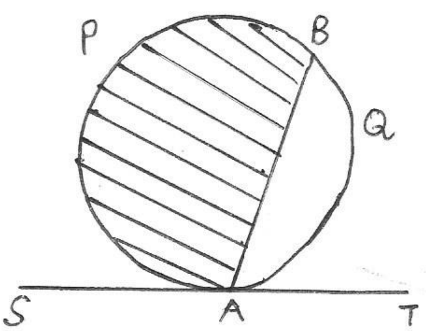
Eg 2. Find the size of angle x.

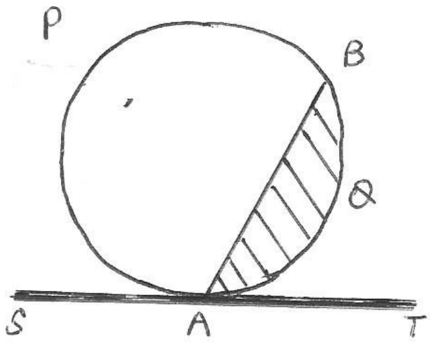


eg. 3 Find the size of angles x and y.



ALTERNATE SEGMENT

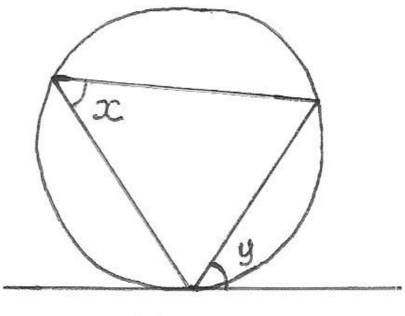




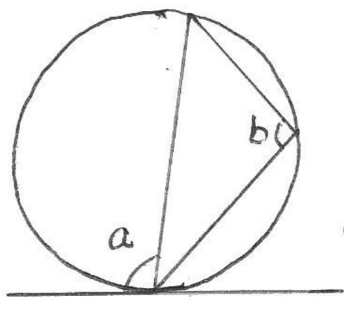
In both figures SAT is a tangent to the circle at A. The chord AB divides the circle into 2 segments. APB is the alternate segment to angle TAB ie it is on the other side of AB from angle TAB.

Similarly segment AQB is the alternate segment to the angle SAB.

If a straight line touches the circle and from the point of contact a chord is drawn, the angle the chord makes with the tangent is equal to the angle in the alternate segment.

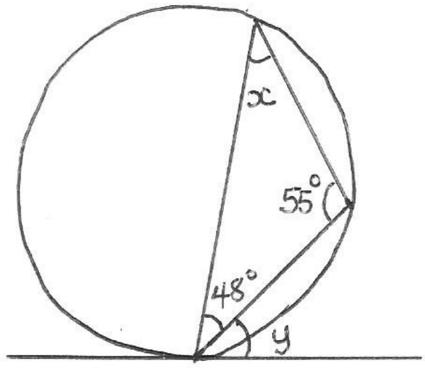


$x = y$



$a = b$

eg. Find the angles marked.



$x + 48° + 55° = 180°$ (Interior angle sum of a triangle)

$x + 103° = 180°$

$x = 77°$

$x = y$ (angle a chord makes with a tangent is equal to the angle in the alternate segment

$\therefore$ $y = 77°$

Try out the number below.
Find the angles marked.